Discrete Mathematics Final, Spring 2018

Professor Callahan

Name: _____

Net ID: _____ Section A or B: ___

40 multiple choice, 3 points each:

1. Consider the following function: what is its Θ run time?

```
f(n):
   for i from 1 to n:
        print i;
        i = floor( i / 2)
```

a. *n*
b. *n* log *n*
*c. log *n*
d. *n* / 2

2. A customer can choose one of 5 monitors, one of 3 keyboards, one of 2 CPUs and one of 4 printers, to buy a computer system. Determine the number of possible systems that the customer can choose from.
a. 14
b. 15
c. 100
*d. 120

4. If Helen has many red, blue, yellow and green shoes and if she decides to blindly select a few shoes, how many shoes must she pull out to guarantee that she has a pair with matching colors?
a. 4
*b. 5
c. 3
d. Cannot be determined

5. A coin is tossed 7 times. What's the probability of getting 7 tails?
a. 1/64
b. 1/7
*c. 1/128
d. 1/32

6. Disha draws two cards out of a deck of 52 cards. What is the probability of drawing a Queen and then a King without replacement?
a. 2/663
*b. 4/663
c. 1/310
d. 2/371

7. In how many different ways can the letters of the word 'REAPING' be arranged, such that the vowels always come together?
a. 360
b. 480
*c. 720
d. 5040

8. Which of the following functions grows the most slowly?
a. $f(x) = x \log x$
b. $f(x) = 7x$
*c. $f(x) = \log x$
d. $f(x) = 7x + 2x^2$

9. Euclid's algorithm for finding the greatest common divisor works because:
a. He was a real smart guy.
b. $ax + by = 1$.
c. The number of primes is infinite.
*d. Whatever divides a and b also divides b and $a - bx$.

10. If $A \cap B = B \cap A$, then:
a. $A = B$
b. B is a subset of A
c. A is a subset of B
*d. This is always true, so it tells us nothing about A or B.

11. Which of these functions is one-to-one from R to R.
*a. $f(x) = x + 5$
b. $f(x) = -3x^2 + 7$
c. $f(x) = -x^2 + 1$
d. Both a and c

12. Which of the following is a tautology?
*a. $(\neg r \wedge (q \rightarrow r)) \rightarrow \neg q$
b. $(r \wedge (q \rightarrow r)) \rightarrow q$
c. $(r \wedge (q \rightarrow r)) \rightarrow \neg q$
d. $(\neg r \wedge (\neg q \rightarrow r)) \rightarrow \neg q$

13. Mathematically, a graph is defined as
a. a non-empty set of edges and a set of vertices.
*b. a non-empty set of vertices and a set of edges.
c. a set of edges and a set of vertices.
d. a collection of lines and dots on paper or a computer screen.

14. How many different 8 letter words are possible if the first and third letters are both 'P'?

a. 26^4
b. 26*25*24*23*22*21*20*19
*c. 26^6
d. 26^2

15. Mathematically speaking, a function
a. takes an input value in its domain and maps it to any output.
*b. takes an input value in its domain and maps it to a unique output.
c. takes an integer and maps it to a real number.
d. accepts some input, but may or may not produce any output.

16. What integers can be "reached" from the equation $22x + 63y$ by choosing appropriate integer values of x and y?
*a. all integers
b. all multiples of 2
c. all multiples of 3
d. all multiples of 7

17. What integers can be "reached" from the equation $21x + 60y$ by choosing appropriate integer values of x and y?
a. all integers
b. all multiples of 2
*c. all multiples of 3
d. all multiples of 8

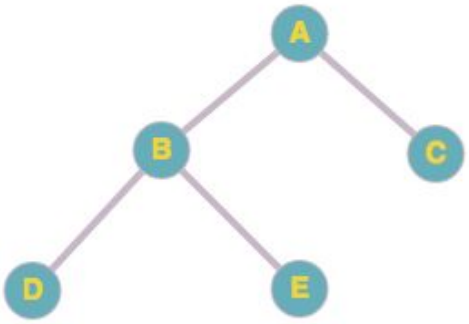
18. Is the number 9 relatively prime?
a. No, it has a prime factor of 3.
*b. The question makes no sense: it is like asking "Is Joe shorter?"
c. Yes.

19. The unique prime factorization of 96 is:
b. 2 * 48
c. 2 * 3
d. 3 * 32
*a. 2 * 2 * 2 * 2 * 2 * 3

20. A solution for the system $x \equiv 3 \pmod 4$ and $x \equiv 5 \pmod 6$ is
a. 36
*b. 23
c. 15
d. There is no solution.

21. To find the maximum value in a binary search tree, we...
*a. can just follow the rightmost path in the tree to a leaf.
b. can just follow the leftmost path in the tree to a leaf.
c. find the successor of the root node.
d. find the successor of the rightmost leaf.

22. Consider the following tree:



Which of the following traversals yield DEBCA?
a. Inorder
b. Preorder
*c. Postorder
d. None of the above

23. If we are part way through building a minimum spanning tree, and we see that we have a forest containing several trees, that means we are running
a. Prim's algorithm
*b. Kruskal's algorithm
c. Either Prim's algorithm or Kruskal's algorithm
d. Can't be Kruskal's or Prim's

24. Which of the following is a valid Huffman coding for an alphabet with letters having the these frequencies:
a: .01, b: .03, c: .09, d: 16, e: .24, f: .47

a. a: 00000, b: 0000, c: 000, d: 001, e: 01, f: 1
b. a: 1, b: 01, c: 001, d: 0001, e: 00001, f: 00000
c. a: 11111, b: 1111, c: 111, d: 11, e: 1, f: 0
*d. a: 00000, b: 00001, c: 0001, d: 001, e: 01, f: 1

25. What sort of traversal does the following code make?

```
XTraversal(tree_root):
        if tree_root == nil:
            return
        else:
            XTraversal(tree_root.left)
```

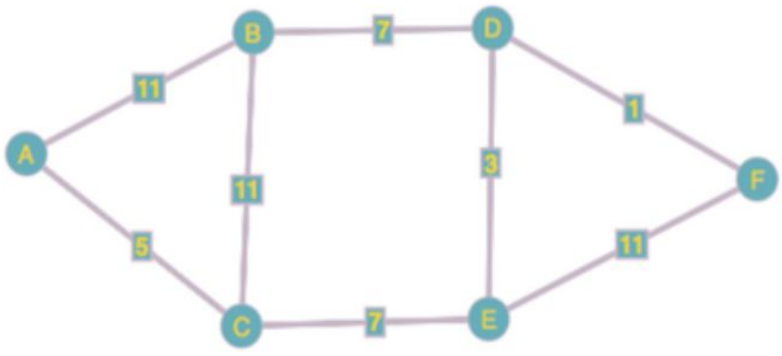
```
print(tree_root)
XTraversal(tree_root.right)
```

*a. Inorder traversal
b. Postorder traversal
c. Preorder traversal
d. Prim's traversal

26. (1 point) What is the value of following prefix expression?
↑ − ∗ 2 4 ∗ 3 2 6
a. 32
b. 8
c. 4
*d. 64

27. A graph has 21 vertices and 20 edges. Might it be a tree?
a. It definitely is.
*b. Could be.
c. No way!
d. Can't say.

28. In finding the shortest path from A to F, what is the first time a path distance is updated by finding a second, shorter path to a vertex?



a. When we discover getting from C to B.

b. When we discover getting from B to D.
*c. When we discover getting from E to D.
d. When we discover getting from E to F.

29. Consider the multigraph with following adjacency matrix:

	a	**b**	**c**	**d**	**e**
a	0	1	1	0	1
b	1	0	0	1	1
c	1	0	0	1	1
d	0	1	1	0	2
e	1	1	1	2	0

ALL ANSWERS CORRECT!

30. Consider the graph described by the adjacency list:
{a: (b, c, d, g), b: (a, c), c: (a, b, d), d: (a, c), e: (g), f: (g), g: (e, f, a)}
Does it contain an Eulerian path?
a. Yes
*b. No
c. Not enough information

31. A connected planar graph having 8 vertices and 12 edges contains _____ faces.
a. 20
b. 4
c. 5
*d. 6

32. There are 21 people at a party. Is it possible for each person to exchange business cards with exactly 5 other people?
a. Yes
*b. No
c. Not enough Information

33. K_9 is the name for:
a. Tandon's dog patrol unit.
*b. The complete graph with 9 vertices.

c. The complete graph with 9 edges.
d. The cycle graph with 9 vertices.

34. A graph is called a _____ if it consists of disconnected trees.
a. worthless graph
b. directed graph
c. multigraph
*d. forest

35. For which of the following combinations of the degrees of vertices would a connected graph have an Eulerian circuit?
a. 1, 2, 0
b. 2, 1, 1
*c. 2, 2, 2
d. All of the above

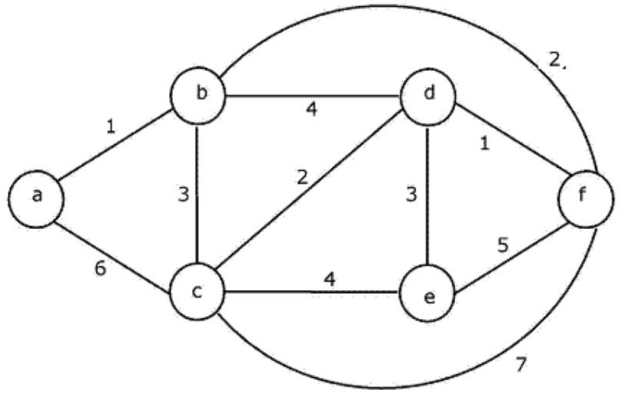
36. Using the master theorem, the runtime complexity of the recurrence $T(n) = 8T(n/2) + n^3$ is:
a. $\Theta(n^2 \log n)$
b. $\Theta(n^2)$
*c. $\Theta(n^3 \log n)$
d. Can't be solved using the master theorem.

37. Using the master theorem, the runtime complexity of the recurrence $T(n) = 5T(2n) + n \log n$ is:
a. $\Theta(n^{2.32})$
b. $\Theta(n)$
c. $\Theta(n^5)$
*d. Can't be solved using the master theorem.

38. All integers greater than 1 are either
a. prime or pseudo-prime
*b. prime or composite
c. composite or perfect numbers
d. even or prime
39. (18, 24) =
*a. 6
b. 2

c. 3
d. 1

40. Which one of the following is a possible sequence of edges added, in order, to a minimum spanning tree using Prim's algorithm?



*a. (a—b), (b—f), (f—d), (d—c), (d—e)
b. (a—b), (d—f), (d—c), (b—c), (d—e)
c. (a—b), (b—d), (d—c), (b—f), (d—e)
d. (a—b), (d—f), (b—f), (d—e), (d—c)

5 long answer questions, 6 points each:

1) Please prove Euler's formula (vertices - edges + faces = 2) for a planar graph using induction.

2) How does discrete probability differ from the distributions studied in a statistics class, such as a Gaussian distribution?

3) Prove that Kruskal's algorithm, in making the greedy choice, must find a minimum spanning tree.

4) Prove that if *a* and *b* are relatively prime integers, then the formula $ax + by$ can "reach" any integer by choosing a suitable combination of x and y.

5) How does discrete mathematics differ from a branch of math like calculus?