

Discrete Mathematics Quiz 13

Name: _____

NYU Net ID: _____

Each question carries one point. Please do all 5 questions on both sides of the paper. Total will be best 4 out of 5.

1.1) A fair coin is flipped 10 times. What is the probability that exactly 4 of the 10 flips come up heads?

- a. $(\frac{10}{4})/10!$
- b. $2^4/2^{10}$
- *c. $(\frac{10}{4})/2^{10}$
- d. $4/2^{10}$

1.2) A team of five kids is randomly chosen from a class of 20 students. An outcome of the selection process is the set of five kids selected. What is the size of the sample space?

- a. 5^{20}
- b. 20^5
- *c. $\binom{20}{5}$
- d. $P(20, 5)$

1.3) A group of seven kids line up in a random order. Each ordering of the kids is equally likely. There are three girls and four boys in the group. What is the probability that all the girls are ahead of all the boys?

- *a. $3!4!/7!$
- b. $\binom{7}{3}/2^7$
- c. $2^4/2^7$
- d. $\binom{7}{3}/7!$

2.1) A red die and a blue die are thrown. What is the probability that the numbers on the two dices are different?

- a. $1/6$
- b. $5/36$
- *c. $5/6$

d. $31/36$

2.2) A fair coin is flipped 10 times. What is the probability that the first flip comes up heads or exactly five flips come up heads (or both)?

- a. $(2^9 + (\frac{10}{5}))/2^{10}$
- b. $(2^9 + 2^5)/2^{10}$
- *c. $(2^9 + (\frac{10}{5}) - (\frac{9}{4}))/2^{10}$
- *d. $(2^9 + (\frac{10}{5}) - (\frac{9}{5}))/2^{10}$

2.3) A red die and a blue die are thrown. What is the probability that the red die comes up 5 or the sum on the two dice is less than four (or both)?

- *a. $1/6 + 1/12$
- b. $1/6 + 1/18$
- c. $1/6 + 1/12 - 1/36$
- d. $1/6 + 1/18 - 1/36$

3.1) An experiment is performed and four events (A, B, C, and D) are defined over the set of all possible outcomes. The probabilities of the four events and their intersections are:

$p(A) = 1/3$	$p(A \cap B) = 1/6$	$p(A \cap C) = 0$
$p(B) = 1/2$	$p(B \cap C) = 1/8$	$p(B \cap D) = 1/8$
$p(C) = 1/4$	$p(C \cap D) = 1/12$	
$p(D) = 1/3$	$p(A \cap D) = 1/8$	

Which pair of states are independent?

- a. A and C
- b. A and D
- *c. B and C
- d. B and D

3.2) An experiment is performed and four events (A, B, C, and D) are defined over the set of all possible outcomes. Use the table below to select the pair of events that are independent:

$p(A) = 1/4$	$p(A C) = 1/3$
$p(B) = 1/4$	$p(A D) = 1/6$
$p(C) = 1/3$	$p(B C) = 1/4$
$p(D) = 1/6$	$p(D B) = 1/4$

- a. A and C
- b. A and D
- *c. B and C
- d. D and B

4.1) A chip company has two manufacturing plants. Plant A produces 40% of the chips and Plant B produces 60% of the chips produced by the company. The company knows that 2% of the chips produced by plant A are defective and 1% of the chips produced by plant B are defective. If a randomly chosen chip produced by the company is defective, what is the likelihood that the chip came from plant A?

- *a. $\frac{.02 \times .4}{(.02 \times .4) + (.01 \times .6)}$
- b. $\frac{.01 \times .6}{(.02 \times .4) + (.01 \times .6)}$
- c. $\frac{.02 \times .6}{(.02 \times .6) + (.01 \times .4)}$
- d. $\frac{.01 \times .4}{(.02 \times .6) + (.01 \times .4)}$

4.2) A blood test is used to determine whether a person has a disease. 1% of the total population has the disease. If a person has the disease, then the probability that the test outcome is positive is .95. If the person does not have the disease, then the probability that the test outcome is negative is .98. If a person takes the blood test and the outcome is positive, then what is the probability that the person has the disease?

- a. $\frac{.99 \times .02}{(.01 \times .95) + (.99 \times .02)}$
- b. $\frac{.99 \times .98}{(.01 \times .95) + (.99 \times .98)}$
- *c. $\frac{.01 \times .95}{(.01 \times .95) + (.99 \times .02)}$
- d. $\frac{.01 \times .95}{(.01 \times .95) + (.99 \times .98)}$

5.1) A fair coin is tossed three times. The random variable X is defined to be 2^h , where h is the number of flips that come up heads. For example, $X(\text{HHT}) = 2^2$. What is $E[X]$?

- a. 1
- b. 3/2
- c. 2
- *d. 27/8

5.2) A blue die and a red die are thrown in a game. If the sum of the two numbers is 7 or 11, the player wins \$10. If the sum of the two numbers is 12, then the player wins \$20. In all other cases, the player loses a dollar. What are the expected winnings of the player in one game?

a. $53/36$

b. $63/36$

*c. $73/36$

d. $100/36$

5.3) A player spins two spinners. The outcome of each spinner is 1, 2, or 3. Each outcome is equally likely. Define the random variable X to be the maximum of the two numbers on the spinners. What is $E[X]$?

a. $2/3$

b. $5/3$

c. $20/9$

*d. $22/9$