Discrete Mathematics Quiz 3

Name: ______________________________

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Each question is worth 1 point.

1.1) **Theorem:** For any real number x, if $x^2 - 6x + 5 > 0$, then $x \geq 5$ or $x \leq 1$.
Which facts are assumed and which facts are proven in a proof by contrapositive of the theorem?

a. Assumed: $x \geq 5$ or $x \leq 1$
Proven: $x^2 - 6x + 5 \leq 0$

b. Assumed: $x \geq 5$ and $x \leq 1$
Proven: $x^2 - 6x + 5 \leq 0$

c. Assumed: $x < 5$ or $x > 1$
Proven: $x^2 - 6x + 5 \leq 0$

*d. Assumed: $1 < x < 5$
Proven: $x^2 - 6x + 5 \leq 0$

1.2) **Theorem:** For any two real numbers, x and y, if x and y are both rational then x + y is also rational.
Which facts are assumed and which facts are proven in a proof by contrapositive of the theorem?

a. Assumed: x is rational or y is rational.
Proven: x + y is rational

b. Assumed: x is rational and y is rational.
Proven: x + y is rational

*c. Assumed: x + y is irrational
Proven: x is irrational or y is irrational

d. Assumed: x + y is irrational
Proven: x is irrational and y is irrational

1.3) **Theorem:** For any real number x, if $0 \leq x \leq 3$, then $15 - 8x + x^2 > 0$
Which facts are assumed and which facts are proven in a proof by contrapositive of the theorem?

a. Assumed: $0 \leq x$ or $x \leq 3$
Proven: $15 - 8x + x^2 > 0$

b. Assumed: $0 \leq x$ and $x \leq 3$

Proven: $15 - 8x + x^2 > 0$

*c. Assumed: $15 - 8x + x^2 \leq 0$

Proven: $x < 0$ or $x > 3$

d. Assumed: $15 - 8x + x^2 \leq 0$

Proven: $0 < x$ and $x > 3$

2.1) The domain of discourse for x and y is the set of employees at a company. Miguel is one of the employees at the company. Define the predicate:

N(x, y): x earns more than y

Select the logical expression that is equivalent to:

"Exactly one person earns more than Miguel."

a) $\exists x \;\; N(x, Miguel)$

b) $\exists x \;\; \forall y \;\; (N(x, Miguel) \land \neg N(y, Miguel))$

*c) $\exists x \;\; \forall y \;\; (N(x, Miguel) \land ((y \neq x) \rightarrow \neg N(y, Miguel)))$

d) $\exists x \;\; \forall y \;\; (N(x, Miguel) \rightarrow ((y \neq x) \rightarrow \neg N(y, Miguel)))$

2.2) The domain of discourse for x and y is the set of employees at a company. Miguel is one of the employees at the company. Define the predicate:

N(x, y): x earns the same or more than y

Select the logical expression that is equivalent to:

"Exactly one person earns less than Miguel."

a) $\exists x \;\; N(x, Miguel)$

b) $\exists x \;\; \forall y \;\; (N(x, Miguel) \land \neg N(y, Miguel))$

*c) $\exists x \;\; \forall y \;\; (N(Miguel, x) \land ((y \neq x) \rightarrow \neg N(Miguel, y)))$

d) $\exists x \;\; \forall y \;\; (N(x, Miguel) \rightarrow ((y \neq x) \rightarrow \neg N(y, Miguel)))$

2.3) The domain of discourse for x and y is the set of employees at a company. Define the predicate:

V(x): x is a manager

M(x, y): x earns more than y

Select the logical expression that is equivalent to:

"Every manager earn more than every employee who is not a manager."

a) $\forall x \;\; \forall y \;\; (M(x, y) \rightarrow (V(x) \rightarrow \neg V(y)))$

b) $\forall x \;\; \forall y \;\; (V(x) \lor \neg V(y) \lor M(x, y))$

*c) $\forall x \;\; \forall y \;\; ((V(x) \land \neg V(y)) \rightarrow M(x, y))$

d) $\forall x \;\; \forall y \;\; M(V(x), \neg V(y))$

3.1) **Theorem:** If r and s are rational numbers, then the product of r and s is a rational number. Which facts are assumed in a direct proof of the theorem?

a) rs = a/b, where a and b are integers $a \neq 0$.

b) rs = a/b, where a and b are integers $b \neq 0$.

c) r = a/b, and s = c/d, where a, b, c, d are integers and $a \neq 0$ and $c \neq 0$.

*d) r = a/b, and s = c/d, where a, b, c, d are integers and $b \neq 0$ and $d \neq 0$.

3.2) **Theorem:** For any two real numbers, x and y, if x and y are both rational then x + y is also rational. Which facts are assumed and which facts are proven in a direct proof of the theorem?

a) Assumed: x is rational or y is rational.
    Proven: x + y is rational

*b) Assumed: x is rational and y is rational.
    Proven: x + y is rational

c) Assumed: x + y is rational
    Proven: x is rational or y is rational

d) Assumed: x + y is irrational
    Proven: x is irrational and y is irrational

3.3) **Theorem:** For any real number x, if $0 \leq x \leq 3$, then $15 - 8x + x^2 > 0$. Which facts are assumed and which facts are proven in a direct proof of the theorem?

a) Assumed: $0 \leq x$ or $x \leq 3$
    Proven: $15 - 8x + x^2 > 0$

*b) Assumed: $0 \leq x$ and $x \leq 3$
    Proven: $15 - 8x + x^2 > 0$

c) Assumed: $15 - 8x + x^2 \leq 0$
    Proven: $0 > x$ or $x > 3$

d) Assumed: $15 - 8x + x^2 \leq 0$
    Proven: $0 > x$ and $x > 3$

4.1) If x is a positive integer less than 4, then $(x+1)^3 \geq 4^x$.
Prove by exhaustion method.
Answer like:

$(1+1)^3 \geq 4^1$

$(2+1)^3 \geq 4^2$

$(3+1)^3 \geq 4^3$

4.2) If x is a positive integer less than 3, then $(2x)^3 \geq 4^{x-1}$.
Prove by exhaustion method.
Answer like:

$(2 * 1)^3 \geq 4^{1-1}$
$(2 * 2)^3 \geq 4^{2-1}$

4.3) If x is a negative integer greater than -5, then $(x-5)^4 \geq 2^{x+1}$.
Prove by exhaustion method.
Answer like:
$(-1-5)^4 \geq 2^{-1+1}$
$(-2-5)^4 \geq 2^{-2+1}$
$(-3-5)^4 \geq 2^{-3+1}$
$(-4-5)^4 \geq 2^{-4+1}$

5) (optional) Match the left side statements with the right side statements.

1) direct proof	a) prove a conditional theorem of the form p → c by showing that ¬c → ¬p
2) proof by contrapositive	b) prove the theorem statement t by making the assumption ¬t and leads to a conclusion r ∧ ¬r, for some proposition r.
3) proof by contradiction	c) breaks the domain for the variable x into different classes and gives a different proof for each class
4) proof by cases	d) prove a conditional statement p → c

1-d,
2-a,
3-b,
4-c