Discrete Mathematics Quiz 4

Name: ____________________________

NYU Net ID: _______________________

1.) Select the set that is equal to: {3, 5, 7, 9, 11, 13 }

a. $\{x : x \in Z \text{ and } 3 < x < 14\}$

b. $\{x : x \in R \text{ and } 3 \leq x < 14\}$

*c. $\{x : x \in Z \text{ and } x \text{ is odd and } 3 \leq x \leq 14\}$

d. $\{x : x \in Z \text{ and } x \text{ is prime and } 3 \leq x < 14\}$

1.2) Use the definitions below to select the statement that is true.

A = {x : x ∈ Z and x is even}

B = {x : x ∈ Z and −4 < x < 17}

a. A is finite.

b. $B \subseteq A$

c. $A \subset A$

*d. A ∩ B = {x : x ∈ Z and x is even and −4 < x < 17}

1.3) Use the definition below to select the statement that is false.

A = {x : x ∈ Z and x is even and 4 < x < 17}

a. $4 \notin A$

b. $6 \in A$

c. $17 \notin A$

*d. $|A| = 7$

2.1) $A = \{x : x \in Z \text{ and } x \text{ is even}\}$ $C = \{3, 5, 9, 12, 15, 16\}$

Select the true statement.

a. C - A = {12, 16}

*b. C - A = {3, 5, 9, 15}

c. C - A = {3, 5, 9, 12, 15}

d. The set C - A is infinite.

2.2) $B = \{x : x \in Z \text{ and } x \text{ is a prime number}\}$ $C = \{3, 5, 9, 12, 15, 16\}$

The universal set U is the set of all integers. Select the set corresponding to $\overline{B} \cap C$.

a. {3, 5}

b. {9, 12, 16}

c. {3, 5, 9, 15}
*d. {9, 12, 15, 16}

2.3) $C = \{3, 5, 9, 12, 15, 16\}$ $D = \{5, 7, 8, 12, 13, 15\}$
Select the set corresponding to $C \oplus D$.
a. {3, 9, 16}
b. {5, 12, 15}
*c. {3, 7, 8, 9, 13, 16}
d. {3, 5, 7, 8, 9, 12, 13, 15, 16}

3.1) A = {a, b} B = {1, 2, 3}
Select the expression that is an element of $B \times A \times B$.
a. (1, 3, b)
*b. (3, b, 1)
c. (b, 3, 1)
d. (b, 1, 3)

3.2) A = {a, b} B = {1, 2, 3}
Select the expression that is an element of $A \times B \times B$.
a. (1, 3, b)
b. (3, b, 1)
*c. (b, 3, 1)
d. (1, b, 3)

3.3) A = {a, b} B = {1, 2, 3}
Select the expression that is an element of $A \times B \times A$.
a. (a, b, 3)
b. (3, b, b)
*c. (b, 3, b)
d. (3, b, 3)

4.1) What is the power set of {1, 2, 3}?
Answer: { $\varnothing$, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3} }

4.2) What is the power set of {a, b, c}?
Answer: { $\varnothing$, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c} }

4.3) What is the power set of {-1, 0, 1}?
Answer: { $\varnothing$, {-1}, {0}, {1}, {-1, 0}, {-1, 1}, {0, 1}, {-1, 0, 1} }

Choose one between 5) and 6)

5) Given the table below

Operation	Notation	Description
Intersection	$A \cap B$	$\{x : x \in A \text{ and } x \in B\}$
Union	$A \cup B$	$\{x : x \in A \text{ or } x \in B \text{ or both}\}$
Difference	$A - B$	$\{x : x \in A \text{ and } x \notin B\}$
Symmetric difference	$A \oplus B$	$\{x : x \in A - B \text{ or } x \in B - A\}$
Complement	$\overline{A}$	$\{x : x \notin A\}$

Use set identity and logic:

5.1) Show that if A and B are sets then $A - B = A \cap \overline{B}$.

5.2) Prove the associative law that if A, B, and C are sets, then $A \cup (B \cup C) = (A \cup B) \cup C$.

5.3) Prove the associative law that if A, B, and C are sets, then $A \cap (B \cap C) = (A \cap B) \cap C$.

Example: to prove $A \cap B = B \cap A$,

$X \in A \cap B$	$\leftrightarrow$	$\{x \mid x \in A \land x \in B\}$	by definition
$\{x \mid x \in A \land x \in B\}$	$\leftrightarrow$	$\{x \mid x \in B \land x \in A\}$	by commutative law
$\{x \mid x \in B \land x \in A\}$	$\leftrightarrow$	$X \in B \cap A$	by definition

Solution of 5)

5.1) Both sides equal $\{x \mid x \in A \land x \notin B\}$.

$A - B = \{x \mid x \in A \land x \notin B\}$ by definition

$A \cap \overline{B} = \{x \mid x \in A \land x \notin B\}$ by definition

So $A - B = A \cap \overline{B}$

5.2) Both sides equal $\{x \mid x \in A \lor x \in B \lor x \in C\}$

$A \cup (B \cup C) = \{x \mid x \in A\} \cup \{x \mid x \in B \lor x \in C\}$ by definition

$= \{x \mid x \in A \lor x \in B \lor x \in C\}$ by definition

$(A \cup B) \cup C = \{x \mid x \in A \lor x \in B\} \cup \{x \mid x \in C\}$ by definition

$= \{x \mid x \in A \lor x \in B \lor x \in C\}$ by definition

So $A \cup (B \cup C) = (A \cup B) \cup C$

5.3) Both sides equal $\{x \mid x \in A \land x \in B \land x \in C\}$

$A \cap (B \cap C) \quad = \{x \mid x \in A\} \cap \{x \mid x \in B \wedge x \in C\}$ by definition

$\quad = \{x \mid x \in A \wedge x \in B \wedge x \in C\}$ by definition

$(A \cap B) \cap C \quad = \{x \mid x \in A \wedge x \in B\} \cap \{x \mid x \in C\}$ by definition

$\quad = \{x \mid x \in A \wedge x \in B \wedge x \in C\}$ by definition

So $A \cap (B \cap C) = (A \cap B) \cap C$

6) Given the table of set identities:

Name	Identities	
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \varnothing = A$	$A \cap U = A$
Domination laws	$A \cap \varnothing = \varnothing$	$A \cup U = U$
Double Complement law	$\overline{\overline{A}} = A$	
Complement laws	$A \cap \overline{A} = \varnothing$ $\overline{U} = \varnothing$	$A \cup \overline{A} = U$ $\overline{\varnothing} = U$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$

prove the following new identities. Label each step in your proof with the set identity used to establish that step.

a) $\overline{A} \cap (A \cup B) = \overline{A} \cap B$

b) $(\overline{A} \cap C) \cup (A \cap C) = C$

c) $\overline{A} \cup (A \cap B) = \overline{A} \cup B$

d) $A = (A \cap B) \cup (A \cap \overline{B})$

For a), b), and c) see text section 3.5 exercise 3.5.2 solutions. Solution of d) is similar to exercise 3.5.2 b.