Discrete Mathematics Quiz 5

Name: ______________________________

NYU Net ID: _________________________

Each question carries one point.

1.1) Select the expression that is equal to $(3^{k+1})^2$

a) 3^{k+2}

b) 3^{k+3}

c) 3^{2k+1}

*d) 3^{2k+2}

1.2) Select the value that is equal to $\lfloor log_2 29 \rfloor$

a) 2

b) 3

*c) 4

d) 5

1.3) Select the expression that is equal to $\frac{log_5 k^3}{log_5 27}$

*a) $log_3 k$

b) $log_5 k$

c) $log_{27} k$

d) $3 log_5 (k/3)$

2.1) Select the Boolean expression that is equivalent to the function defined in the table below:

x	y	z	f(x, y, z)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0

1	1	1	0

a) $\bar{x}y\bar{z} + \bar{x}yz$

*b) $\bar{x}y\bar{z} + \bar{x}yz + x\bar{y}z$

c) $\bar{x}y\bar{z} + xyz + x\bar{y}\ \bar{z}$

d) $\bar{x}y\bar{z} + \bar{x}yz + x\bar{y}z + xyz$

2.2) Select the Boolean expression that is equivalent to the function defined in the table below:

x	y	z	f(x, y, z)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

a) $x\overline{yz} + xyz$

b) $x\bar{y}\ \bar{z} + \bar{x}\ \bar{y}\ \bar{z}$

*c) $x\bar{y}\ \bar{z} + xyz$

d) $x\overline{yz} + \overline{xyz}$

2.3) Select the Boolean expression that is equivalent to the function defined in the table below:

x	y	z	f(x, y, z)
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

*a) $\bar{x}\ \bar{y}\ \bar{z} + \bar{x}y\bar{z} + xyz$

b) $\bar{x}\ \bar{y}\ \bar{z} + x\bar{y}z + xyz$

c) $\overline{xyz} + x\bar{y}z + xyz$

d) $\overline{xyz} + \bar{x}y\bar{z} + xyz$

3.1) Select the expression that is equivalent to $(x+\overline{y})z$

a) $\overline{\overline{x+y}+\overline{z}}$

b) $\overline{\overline{x+\overline{y}}+z}$

c) $\overline{x+\overline{y}+\overline{z}}$

*d) $\overline{\overline{x+\overline{y}}+\overline{z}}$

3.2) Select the Boolean expression that is equivalent to x + y.

a) $(x\uparrow x)\downarrow(y\uparrow y)$

*b) $(x\uparrow x)\uparrow(y\uparrow y)$

c) $(x\uparrow y)\downarrow(x\uparrow y)$

d) $(x\downarrow x)\downarrow(y\downarrow y)$

3.3) Select the Boolean expression that is equivalent to xy.

a) $(x\downarrow y)\uparrow(x\downarrow y)$

b) $(x\uparrow x)\uparrow(y\uparrow y)$

c) $(x\downarrow y)\downarrow(x\downarrow y)$

*d) $(x\uparrow x)\downarrow(y\uparrow y)$

4.1) Select the Boolean expression that is not satisfiable.

a) $(x+y)(x+z)(y+\overline{z})$

*b) $(\overline{x+y})(x+z)(y+\overline{z})$

c) $(\overline{x}+\overline{y})(x+z)(y+\overline{z})$

d) $(\overline{xy})(x+z)(y+\overline{z})$

4.2) Consider a school with two periods during which classes can be scheduled. For each class, there are two variables. For example, for class A, there are variables x_{A1} and x_{A2}. Setting $x_{A2}=1$ represents scheduling class A during period 2. Select the Boolean expression that is true if and only if class A is scheduled during exactly one of the two periods.

a) $(x_{A1}x_{A2})(\overline{x_{A1}}+\overline{x_{A2}})$

b) $(x_{A1}+x_{A2})(\overline{x_{A1}}\ \overline{x_{A2}})$

*c) $(x_{A1}+x_{A2})(\overline{x_{A1}x_{A2}})$

d) $(x_{A1}x_{A2})(\overline{x_{A1}+x_{A2}})$

4.3) Consider a school with two periods during which classes can be scheduled. For each class, there are two variables. For example, for class A, there are variables x_{A1} and x_{A2}. Setting $x_{A2} = 1$ represents scheduling class A during period 2. Select the Boolean expression that is true if and only if classes A and B are not scheduled during the same period.

a) $(\overline{x_{A1} + x_{B1}})(\overline{x_{A2} + x_{B2}})$

*b) $(\overline{x_{A1}x_{B1}})(\overline{x_{A2}x_{B2}})$

c) $(\overline{x_{A1} + x_{A2}})(\overline{x_{B1} + x_{B2}})$

d) $(\overline{x_{A1}x_{A2}})(\overline{x_{B1}x_{B2}})$

5.1) Suppose that f is a function from A to B and g is a function from B to C, show that if both f and g are one-to-one functions, then g ∘ f is also one-to-one.

Suppose that a1∈A and a2∈A for which g ∘ f(a1) = g ∘ f(a2).
Then by definition of composition of functions g(f(a1)) = g ∘ f(a1) = g ∘ f(a2) = g(f(a2)).
Since g : B → C is a one–to–one function and f(a1)∈B and f(a2)∈B and g(f(a1)) = g(f(a2)), it follows that f(a1) = f(a2).
Since f : A → B is a one–to–one function and a1∈A and a2∈A and f(a1) = f(a2), it follows that a1 = a2.
Therefore, if g ∘ f(a1) = g ∘ f(a2) then a1 = a2.
By the definition of one–to–one g ∘ f is one–to–one.

5.2) Suppose that f is a function from A to B and g is a function from B to C, show that if both f and g are onto functions, then g ∘ f is also onto.

For every c∈C, ∃ b∈B such that c=g(b), by surjectivity (onto) of g.
For that same b, ∃ a∈A such that b=f(a), by surjectivity of f.
So for every c∈C, ∃ a∈A such that c=g(f(a)) = g∘f(a), i.e., g∘f is (onto)

5.3) Suppose that f is a function from A to B and g is a function from B to C, show that if g ∘ f is onto then g is also onto.

Since g ∘ f is onto, for c∈C, ∃a∈A such that g ∘ f(a) = g(f(a)) = c.
Let f(a) = b, b∈B, and g(b) = c
So, for any c∈C, ∃b∈B such that g(b) = c,
Therefore, g is onto.

5.4) Suppose that f is a function from A to B and g is a function from B to C, show that if $g \circ f$ is one-to-one then f is also one-to-one.

Suppose f is not one to one, then there must be $x,y \in A$, $x \neq y$ such that $f(x)=f(y)$ and therefore $g(f(x))=g(f(y))$ but this means $g \circ f$ is not one to one and this is a contradiction. So f must be one to one.