Discrete Mathematics Quiz 7

Name: _____

NYU Net ID: _____

1.1) Select the function that is $\Theta(n \log n)$.

a) $5n + 17 \log n$

b) $6 n \log n + n^{1.1} + 2$

*c) $23 n \log \log n + 3 n \log n$

d) $2^n \log n + n$

1.2) Select the function that is **not** $\Omega(n^2)$

a) 2^n

b) $n \log n + 3 n^2$

c) $n!$

*d) $n \log n + \log \log n$

1.3) Select the function that is $O(2^n)$

a) 3^n

b) $n!$

*c) n^2

d) $n \log n + 4^n$

2.1) Select the asymptotic worst-case time complexity of the following algorithm:

Algorithm
Input: a1, a2,...,an, a sequence of numbers
n, the length of the sequence
x, a number
Output: ??

```
i := 1
While (x^2 ≠ a_i and i < n)
    i := i + 1
End-while
If (x^2 = a_i) Return( i )
```

Return(-1)

a) $\Theta(1)$
*b) $\Theta(n)$
c) $\Theta(n^2)$
d) $\Theta(n^3)$

2.2) Select the asymptotic worst-case time complexity of the following algorithm:
Algorithm
Input: a1, a2,...,an, a sequence of numbers
n, the length of the sequence
x, a number
Output: ??

For i = 1 to 3
If ($a_i < x$) Return("True")
End-for
Return("False")

*a) $\Theta(1)$
b) $\Theta(n)$
c) $\Theta(n^2)$
d) $\Theta(n^3)$

3.1) Select the asymptotic worst-case time complexity of the following algorithm:
Algorithm
Input: a1, a2,...,an, a sequence of numbers
n, the length of the sequence
x, a number
Output: ??

For i = 1 to n-1
For j = i+1 to n
For k = 1 to n
If ($(a_i)^2 + (a_j)^2 = (a_k)^2$) Return("True")
End-for
End-for
End-for
Return("False")

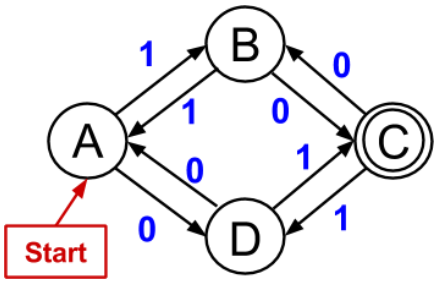
a) $\Theta(1)$
b) $\Theta(n)$
c) $\Theta(n^2)$
*d) $\Theta(n^3)$

3.2) Select the asymptotic worst-case time complexity of the following algorithm:
Algorithm
Input: a1, a2,...,an, a sequence of numbers
n, the length of the sequence
x, a number
Output: ??

For i = 1 to n-1
For j = i+1 to n
If ($|a_i - a_j| > 0$) Return(“True”)
End-for
End-for
Return(“False”)

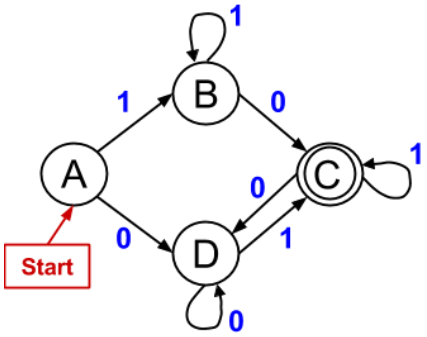
a) $\Theta(1)$
b) $\Theta(n)$
*c) $\Theta(n^2)$
d) $\Theta(n^3)$

4.1) Select the sentence that correctly describes the set of strings accepted by the FSM below:



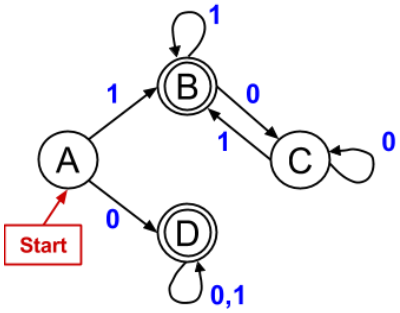
a) The FSM accepts a string x if and only if x contains at least one 0 and at least one 1.
b) The FSM accepts a string x if and only if x starts with a 0 and ends with a 1.
*c) The FSM accepts a string x if and only if the number of 1’s in x is odd and the number of 0’s in x is odd.
d) The FSM accepts a string x if and only if x contains the pattern 101 somewhere in the string.

4.2) Select the current state after the FSM below has processed the string 11010.



a. A
b. B
c. C
*d. D

4.3) Select the current state after the FSM below has processed the string 11011.



a. A
*b. B
c. C
d. D

5.1) Prove for the functions: f(n) = n^2 + n + 3, g(n) = n^2, f(n) is O(g(n))

Select c = 5 and n_0 = 1. We will show that for any n ≥ 1, f(n) ≤ 5·g(n).

$$f(n) = n^2 + n + 3 \leq 5n^2$$

For n ≥ 1, n ≤ n^2 and 1 ≤ n^2, so

$$n^2 + n + 3 \leq n^2 + n^2 + 3n^2$$

Finally, $n^2 + n^2 + 3n^2 = 5n^2 = 5 \cdot g(n)$. Putting the inequalities together, we get that for any n ≥ 1,

$$f(n) = n^2 + n + 3 \leq 5\, n^2 = 5 \cdot g(n)$$

and therefore, $f(n) \leq 5 \cdot g(n)$. f(n) is O(g(n))

5.2) Prove for the functions: $f(n) = n^2 + n + 3$, $g(n) = n^2$, f(n) is $\Omega(g(n))$

Select c = 1 and $n_0 = 1$. We will show that for any $n \geq 1$, $f(n) \geq 1 \cdot g(n)$.

Since $n \geq 1$, $n \geq 0$. Adding the inequalities $n \geq 0$ and $3 \geq 0$ gives that

$$n + 3 \geq 0$$

Add n^2 to both sides to get

$$n^2 + n + 3 \geq n^2$$

Therefore, for $n \geq 1$, $f(n) \geq 1 \cdot g(n)$. f(n) is $\Omega(g(n))$