1.

n and m is positive and even

There exist positive integers a and b that

2a=n and 2b=m (even number definition)

n*m = 2a*2b = 4ab

(n*m) / 4 = 4ab/4 =ab← (ab is integer because a and b are both integers)

so n*m is divisible by 4.

2.Proof by induction:

(1)Base case:

a→*b* or *a*←*b* (only two teams, *a* defeats *b* or *b* defeats *a*)

Base case satisfied the condition of visiting each vertex exactly once (a Hamiltonian path).

(2) Inductive hypothesis: For any tournament of *n* players with a Hamiltonian path, we can always insert the *n* + 1 player and still have a Hamiltonian path.

Suppose *b* → *c* → *d* is a tournament has hamiltonian path, there are three cases:

① a → b → c → d

a defeated *b*, so we can insert *a* at the front of the path.

② b → c → d → a

a was defeated by *d*, so we can insert *a* at the end.

③ b → a → c →d

a lost to *b* but defeated at least one player. Insert *a* before that player. (In the above diagram, that player is *c*.)

3.

Proof:

$(p \lor (p \land \neg F)) \to (q \land (q \lor p)) \equiv p \to q$

Step 1:

$(p \lor (p \land T)) \to (q \land (q \lor p)) \equiv p \to q$ (complement law)

Step 2:

$(p \lor p) \to (q \land (q \lor p)) \equiv p \to q$ (identity law)

Step 3:

$p \to (q \land (q \lor p)) \equiv p \to q$ (Idempotent law)

Step 4:

$p \to q$ (absorption law)

Note: Not the only way to solve this problem. As long as correct laws are applied and logic is correct then people get credits. Valid truth table get credits too.

4.

zxwbkqpr (Not the only answer)
Rules:
① Z must come first
② q must come after Z, x, w, k
③ p, r must come after q
④ k must come after x
⑤ b can be anywhere after z