Discrete Mathematics: Midterm Test with Answers

Professor Callahan, section (A or B): _____

Name: ____________________________ NetID: ____________________

30 multiple choice, 3 points each:

1. If f is defined recursively by:
f (0) = -2,
f (1) = 1, and for n > 1 is f(n) = f(n - 2) + (1 / f(n - 1))
Then f(4) = ______
a. 0
b. -1
c. -2
*d. Not calculable

2. Your street has 13 houses. If 63 people live on your street, what is the minimum number of houses containing at least 5 people?
a. 0
*b. 1
c. 11
d. 12

3. What is the cardinality of the power set of the empty set?
a. Zero
b. Three
c. Two
*d. One

4. Which of the following is ***not*** a logical equivalence?
a. $\neg(p \rightarrow q) \equiv p \land \neg q$
*b. $\neg(p \lor (\neg p \land q)) \equiv \neg p \land q$
c. $(p \rightarrow q) \land (p \rightarrow r) \equiv p \rightarrow (q \land r)$
d. $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

5. Take a octahedral die and roll it twice. What is the probability that the number on the first roll is greater than the number on the second roll? Note: An octahedral die has eight sides, numbered 1 through 8.
a. 1/4
b. 1/3
c. 1/2
*d. 7/16

6. Which of the following is always true (per the associative law)?
*a. $(A \cap B) \cap C = A \cap (B \cap C)$
b. $A \cap (A \cup B) = B$
c. $A \cap (B \cup C) = (A \cap B) \cap (A \cap C)$
d. $A \cap B = B \cup A$

7. In how many different ways can the letters of the word 'SOFTWARE' be arranged in such a way that the letters 'OAE' always come together, in that order?
*a. 720
b. 40,320
c. 4320
d. 16,777,216

8. Which of the following logical expression is the translation of the English sentence
"It is a nice day; and if it is cloudy then it will rain."

p = It is a nice day
q = It is cloudy
r = It will rain

a. $((p \wedge q) \rightarrow r))$
b. $(p \wedge (q \rightarrow \neg r))$
*c. $(p \wedge (q \rightarrow r))$
d. $(q \wedge (r \rightarrow q))$

9. If we represent a set with members drawn from an orderable universe with a bit string that has a 1 in position j if the jth element of the universe is in the set, and a 0 otherwise, then what bit string represents the subset of all odd integers in {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}?
a. 11 1111 1111

*b. 10 1010 1010
c. 01 0101 0101
d. 00 0000 0000

10. The Hebrew alphabet contains 22 letters and has no upper or lower cases. John wants to make a list of passwords he can use in the future, with each password containing a sequence of 3 Hebrew-alphabet letters followed by 2 digits. How many different passwords can John create?
*a. 1,064,800
b. 100,000
c. 1,757,600
d. 33,554,432

11. As mentioned, the Hebrew alphabet contains 22 letters and has no case. Joan wants to make a list of passwords she can use in the future, with each password containing any combination of five Hebrew-alphabet letters and digits. How many different passwords can Joan create?
a. 1,064,800
b. 100,000
c. 1,757,600
*d. 33,554,432

12. Let *T(a, b)* mean that student *a* and *b* play tennis against each other (and no one can play tennis against themselves), and *x, y*, *z* are members of the set of students in school, then which of the following expresses that there are at least three different students who play tennis?
*a $\exists x \exists y \exists z(T(x, y) \wedge T(x, z) \wedge (y \mathrel{!}= z))$
b $\exists x \exists y \exists z(T(x, y) \wedge T(x, z) \wedge (y = z))$
c $\exists x \exists y \forall z(T(x, y) \wedge T(y, z) \wedge (y \mathrel{!}= z))$
d $\exists x \exists y \exists z(T(x, y) \wedge T(x, z) \wedge (x = z))$

13. What is the probability of selecting 4 straight hearts from a deck of 52 cards if each card is ***not*** replaced before the next one is selected? (13 out of the 52 cards are hearts.)
a. one in a billion
b. 1/4
*c. .0026
d. 1/256

14. Given the terms a_1=18 and a_2=54, what is a_3 in a geometric sequence?
*a. 162
b. 72
c. 90
d. 100

15. What is the Big-O runtime of the following function:

```
Procedure1 : removeDuplicates(A)
 for i = 1 to A.length
    for j = i + 1 to A.length
        if A[i] == A[j]
           remove(A[j])
return A
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*Assume remove to be an built-in function in language of your choice.

a. O(n)
*b. O(n^2)
c. O(n log n)
d. O(n)

16. What is the big-O of f(x) = x^3 * log x + x^2.2
a. x^3
*b. x^3 log x
c. x^2.2
d. All of the above.

17. If f(x) = 148x^2, then f(x) is (*restricting* ourselves to the tightest bound):
a. O(148x)
*b. O(x^2)
c. O(x^4)
d. All of the above.

18. Indirect proofs often make use of the following:
a. $p \wedge p \Leftrightarrow p$
b. $p \rightarrow q \Leftrightarrow \sim p \vee q$
c. $p \vee T \Leftrightarrow T$
*d. $p \rightarrow q \Leftrightarrow \sim q \rightarrow \sim p$

19. Consider the sequence 9, 9, 9, 9, 9... which one of the following is true?
a. The sequence is only arithmetic.
b. The sequence is neither arithmetic nor geometric.
c. The sequence is only geometric.
*d. The sequence is both arithmetic and geometric.

20. Here are a few rows of Pascal's triangle:

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

Using these, how many ways can we choose combinations of two of the four suits of cards and three of the five Platonic solids?
a. 40
b. 30
*c. 60
d. 20

21. You live in a place where it is only cloudy one day out of 100, and it is only below 90 F one day out of 100. The odds of both occurring on the same day are most likely:
a. 1 in 10,000, using the product rule
b. 1 in 200, using the sum rule
*c. lower than 1 in 10,000, since most likely the events are not independent

22 Which of the following is a tautology?
a. $(r \land (q \to r)) \to q$
c. $(r \land (q \to r)) \to \neg q$
*c. $(p \land q) \to (p \lor q)$
d. $(\neg r \land (\neg q \to r)) \to \neg q$

23. You roll a die three times. What are the odds that you get at least one result divisible by three?
a. 1/9
*b. 19/27
c. 1/3
d. 8/27

24. What will be the correct recursive formula for the following sequence :

3, 4, 6, 9, 13, 18, 24, 31, 39, . . .

a. $a_1 = 3$; $a_n = a_{n-1} + (n - 3)$.
b. $a_1 = 3$; $a_n = a_{n-1} - (n - 1)$.
c. $a_1 = 1$; $a_n = a_{n-1} + (n - 3)$.
*d. $a_1 = 3$; $a_n = a_{n-1} + (n - 1)$.

25. Consider the following statement:
"All lions are fierce."
"Some lions do not drink coffee."
"Some fierce creatures do not drink coffee."

Let $P(x)$ = "x is a lion"
$Q(x)$ = "x is fierce"
$R(x)$ = "x drinks coffee"
Assuming that the domain consists of all creatures, express the statements in the argument using quantifiers and $P(x)$, $Q(x)$, and $R(x)$.

a. $\forall x(P(x) \rightarrow Q(x))$
$\exists x(P(x) \rightarrow \neg R(x))$
$\exists x(Q(x) \rightarrow \neg R(x))$

*b. $\forall x(P(x) \rightarrow Q(x))$
$\exists x(P(x) \land \neg R(x))$
$\exists x(Q(x) \land \neg R(x))$

c. $\forall x(P(x) \rightarrow Q(x))$
$\exists x(P(x) \rightarrow \neg R(x))$
$\exists x(Q(x) \land \neg R(x))$

d. $\forall x(P(x) \rightarrow Q(x))$
$\exists x(P(x) \land \neg R(x))$
$\exists x(Q(x) \rightarrow \neg R(x))$

26. Which of the following functions grows the most slowly?
a. $f(x) = x \log x$
b. $f(x) = 7x$
c. $f(x) = x^2$
*d. $f(x) = \log x$

27. If for some n and some r, $C(n, r) = 748$, then $C(n, n - r) = ?$
a. $n - 748$
*b. 748
c. $r - 748$
d. not enough information

28. If we are not looking for the tightest bound, then $f(x) = 12$ is:
a. $O(1)$
b. $O(x)$
c. $O(x^2)$
*d. all of the above

29. Cain has 3 children and two of them are boys. What is the probability that the other child is also a boy?
You can assume that there is an equal likelihood of boy or girl children.
*a. 0.25
b. 0.5
c. 0.125
d. 0.33..

30. The cardinality of the powerset $A = \{8, 7, 5, 1\}$ is:
*a. 16
b. 32
c. 4
d. 8

2 long answers, 5 points each:

1. In a round-robin tournament, every player plays every other player. There is a "Hamiltonian path" through the tournament in the case that we can trace a path from player *x* to player *y* to player *z* where *x* --beats--> *y* --beats--> *z*, and so on throughout the whole tournament. Use a proof by induction to show that *every* tournament, with however many players, always has a Hamiltonian path.

2. Show, using a proof by contradiction, that the square root of two is not a rational number.