Discrete Mathematics Fall 2018 Midterm Exam
Prof. Callahan

Section: ________  NetID:    __________   Name:   ____________________________

Multiple Choice Question (30 questions in total, 4 points each)

1 Consider the following propositions:
f: The student got an A on the final.
h: The student turned in all the homework.
p: The student is on academic probation
Select the logical expression that represents the statement: “The student is not on academic probation and the student got an A on the final or turned in all the homework.”
*a. ¬p \land (f \lor h)
 b. (¬p \land f) \lor h
c. ¬p \land f \land h
d. ¬(p \land f) \lor h

2 Select the statement that is false.
a. If 3 is a prime number, then 5 is a prime number.
b. If 4 is a prime number, then 6 is a prime number.
c. If 4 is a prime number, then 5 is a prime number.
*d. If 3 is a prime number, then 6 is a prime number.

3 Select the proposition that is a contradiction.
*a. ¬(p \lor q) \land p
 b. (p \lor q) \land p
c. (¬p \land q) \rightarrow p
d. (¬p \land q) \rightarrow p

4 Translating the following statements into logical expressions:
There is a clever student that fails the test.
C(x): x is a clever student
F(x): x fails the test

a. \forall x (C(x) \land F(x))
b. \forall x (C(x) \rightarrow F(x))
c. \exists x (C(x) \rightarrow F(x))
*d. \exists x (C(x) \land F(x))
5 Select the logical expression that is equivalent to: \( \neg \forall x \ \exists y \ (P(x) \land Q(x,y)) \)

a. \( \exists x \ \forall y \ (\neg P(x) \lor \neg Q(x,y)) \)

b. \( \exists y \ \forall x \ (\neg P(x) \lor \neg Q(x,y)) \)

c. \( \forall y \ \exists x \ (\neg P(x) \lor \neg Q(x,y)) \)

d. \( \forall x \ \exists y \ (\neg P(x) \lor \neg Q(x,y)) \)

6 Theorem: A group of 5 kids has a total of 12 chocolate bars. Then at least one of the kids has at least three chocolate bars.
A proof by contradiction of the theorem starts by assuming which fact?

a. All the kids have three or fewer chocolate bars.

*b. All the kids have less than three chocolate bars.

c. There is a kid with three or fewer chocolate bars.

d. There is a kid with \( r \) fewer than three chocolate bars.

7 Which statement is the contrapositive of: “If \( x = 4 \), then \( 3x = 12 \).”

a. If \( x = 4 \), then \( 3x = 12 \).

b. If \( 3x = 12 \), then \( x = 4 \).

c. If \( x \neq 4 \), then \( 3x \neq 12 \).

*d. \( 3x \neq 12 \), then \( x \neq 4 \).

8 \( A = \{ x \in \mathbb{Z} : x \text{ is even} \} \)
\( C = \{ 3, 5, 9, 12, 15, 16 \} \)
\( D = \{ 5, 7, 8, 12, 13, 15 \} \)

Select the set corresponding to \( C - (A \oplus D) \).

a. \( \{ 3, 9, 16 \} \)

*b. \( \{ 3, 9, 12 \} \)

c. \( \{ 3, 5, 9, 15 \} \)

d. \( \{ 3, 7, 8, 9, 13, 16 \} \)

9 What is the cardinality of the power set of \( \{ \emptyset, 1, \{2,3\} \} \)?

a. 6

b. 7

*c. 8

d. 9

10 Which of the following functions( \( \mathbb{Z} \rightarrow \mathbb{Z} \) ) is onto (surjective)?

a. \( f(x) = 4/x \)

b. \( f(x) = 2x \)

*c. \( f(x) = \lfloor (x+1)/2 \rfloor \)

d. \( f(x) = |x|^3 \)
11 Determine which of the following $f$ is a function from $\mathbb{Z}$ to $\mathbb{R}$
   a. $f(n) = \pm (2n + 3)$
   b. $f(n) = \log_2(n)$
   c. $f(n) = 1 / (n^2 - 1)$
   *d. $f(n) = n / (n^2 + 1)$

12 Determine which of these functions is a bijection (one-to-one & onto) from $\mathbb{R}$ to $\mathbb{R}$.
   *a. $f(x) = 3x^3 + 4$
   b. $f(x) = -3x^2 + 7$
   c. $f(x) = x^3 + 1$
   d. $f(x) = \lceil x/2 \rceil$

13 Select the expression that is equivalent to $\overline{x}$.
   a. $(x + \overline{x})(y + x)$
   b. $\overline{x} y + x$
   c. $(\overline{x} + y)\overline{y}$
   *d. $\overline{xy} + \overline{x}$

14 Select the description that characterizes the Boolean expression: $\overline{xy}$
   *a. Neither CNF nor DNF
   b. CNF, but not DNF
   c. DNF, but not CNF
   d. CNF and DNF

15 Which of the following boolean equation is not satisfiable?
   a. $(x + y)(x + z)(y + \overline{z})$
   *b. $(\overline{x} + \overline{y})(x + z)(y + \overline{z})$
   c. $(\overline{x} + \overline{y})(x + z)(y + \overline{z})$
   d. $(\overline{xy})(x + z)(y + \overline{z})$

16 Select the function that is $\Theta(n \log n)$.
   a. $5n + 17 \log n$
   b. $6n \log n + n^{1.1} + 2$
   *c. $23n \log(\log n) + 3n \log n$
   d. $2^n \log n + n$

17 Select the Boolean expression that corresponds to the output of the Boolean circuit below:
18 Select the set that corresponds to the relation given in the arrow diagram below:

a. \( \{ (A, 1), (B, 3), (B, 4), (D, 3), (D, 4) \} \)
b. \( \{ (A, 1), (B, 3), (B, 2), (D, 3), (D, 4) \} \)
c. \( \{ (1, B), (3, A), (3, D), (4, B), (4, D) \} \)
d. \( \{ (1, B), (2, B), (3, A), (3, D), (4, D) \} \)

19 Recall that:

The relation \( R \) is **reflexive** if for every \( x \in A \), \( xRx \).

The relation \( R \) is **symmetric** if for every \( x, y \in A \), \( xRy \) implies that \( yRx \).

The domain of a relation \( R \) is the set of integers. \( xRy \) if \( y = x^2 \). Select the description that accurately describes relation \( R \).

a. Reflexive
b. Anti-reflexive
c. Symmetric
*d. Anti-symmetric

20 Graph \( G \) is defined by the arrow diagram below.

Select the pair of vertices such that there is no walk of length 4 in \( G \) from the first vertex to the second vertex.
a. 1, 3
*b. 1, 4
  c. 2, 1
  d. 4, 3

21 S and T are binary relations on the set {a, b, c, d} and are defined by the arrow diagrams below:

Select the pair that is not in S o T.
a. (1, 3)
b. (2, 3)
  *c. (2, 4)
d. (4, 3)

22 Which relation on the set {1, 2, 3, 4} is a partial order?
  a. { (1, 2), (2, 3), (1, 3), (4, 3) }
  b. { (1, 2), (2, 3), (1, 3), (3, 4) }
  c. { (1, 2), (2, 3), (1, 3), (3, 4), (1, 1), (2, 2), (3, 3), (4, 4) }
  *d. { (1, 2), (2, 3), (1, 3), (4, 3), (1, 1), (2, 2), (3, 3), (4, 4) }

23 Which of the following is incorrect about strict orders?
  a. A relationship xRy is called strict order if it’s denoted by x ≺ y
  b. A relation R is a strict order if R is transitive and anti-reflexive
  c. A strict order is basically a partial order without the self-loop
  *d. Strict order doesn’t need to be anti-symmetric, while a partial order does

24 The domain of relation R is the set of all integers. xRy if |x − y| ≤ 1. Which statement correctly characterizes the relation R?
  a. R is an equivalence relation.
  b. R is not an equivalence relation because R is not reflexive.
  c. R is not an equivalence relation because R is not symmetric.
  *d. R is not an equivalence relation because R is not transitive.

25 A = {a, b, c, d} X = {1, 2, 3, 4}.
Each choice defines a function whose domain is A and whose target is X. Select the function that has a well-defined inverse.
  a. f = {(a, 3), (b, 4), (c, 3), (d, 4)}
  b. f = {(a, 3), (b, 3), (c, 3), (d, 3)}
  *c. f = {(a, 3), (b, 4), (c, 2), (d, 1)}
d. \( f = \{(a, 3), (b, 4), (c, 2), (d, 4)\} \)

26 Select the asymptotic worst-case time complexity of the following algorithm:

Input: \( A = [a_1, a_2, ..., a_n] \), an array of ints.
\( n \): the length of the sequence

For \( i = 1 \) to \( n \)
\[ \text{If}( A[i] < x ) \text{ Return(“True”)} \]
End-for
Return( “False” )

a. \( \Theta(1) \)
* b. \( \Theta(n) \)
 c. \( \Theta(n^2) \)
 d. \( \Theta(n^3) \)

27 \( A = \{1, 2, \{3, 4\}, \{5, 6, 7\}\} \), select the statement that is true.

a. \( \{3\} \subseteq A \)
 b. \( \{3, 4\} \subseteq A \)
* c. \( \{1, 2\} \subseteq A \)
 d. \( \{1, 2\} \in A \)

28 For the function \( n^k \) and \( c^n \), what is the asymptotic relationship between these functions?
Assume \( k \geq 1, c > 1 \) and \( k, c \) are constants
*a. \( n^k = O(c^n) \)
 b. \( n^k = \Omega(c^n) \)
 c. \( c^n = \Theta(n^k) \)
 d. \( c^n = O(n^k) \)

29 Select the set that is equivalent to \((B \cap C) \cup \emptyset\).

a. \( \emptyset \)
 b. \( B \)
 c. \( C \)
* d. \( B \cap C \)

30 The predicate \( T \) is defined as:
\[ T(x, y, z): (x + y)^2 = z \]
Select the proposition that is true.

a. \( T(4, 1, 5) \)
* b. \( T(4, 1, 25) \)
c. $T(1, 1, 1)$

d. $T(4, 0, 2)$