Discrete Mathematics Fall 2018 Midterm Exam
Prof. Callahan

Section: _____ NetID: _____ Name: _____

Multiple Choice Question (30 questions in total, 4 points each)

1 Consider the following propositions:
f: The student got an A on the final.
h: The student turned in all the homework.
p: The student is on academic probation
Select the logical expression that represents the statement: "The student is <u>not</u> on academic probation and the student got an A on the final or turned in all the homework."
*a. $\neg p \wedge (f \vee h)$
b. $(\neg p \wedge f) \vee h$
c. $\neg p \wedge f \wedge h$
d. $\neg(p \wedge f) \vee h$

2 Select the statement that is false.
a. If 3 is a prime number, then 5 is a prime number.
b. If 4 is a prime number, then 6 is a prime number.
c. If 4 is a prime number, then 5 is a prime number.
*d. If 3 is a prime number, then 6 is a prime number.

3 Select the proposition that is a contradiction.
*a. $\neg(p \vee q) \wedge p$
b. $(p \vee q) \wedge p$
c. $(\neg p \wedge q) \leftrightarrow p$
d. $(\neg p \wedge \neg q) \rightarrow p$

4 Translating the following statements into logical expressions:
There is a clever student that fails the test.
C(x): x is a clever student
F(x): x fails the test

a. $\forall x \ (C(x) \wedge F(x))$
b. $\forall x \ (C(x) \rightarrow F(x))$
c. $\exists x \ (C(x) \rightarrow F(x))$
*d. $\exists x \ (C(x) \wedge F(x))$

5 Select the logical expression that is equivalent to: $\neg \forall x \ \exists y \ (P(x) \wedge Q(x,y))$

*a. $\exists x \ \forall y \ (\neg P(x) \vee \neg Q(x,y))$

b. $\exists y \ \forall x \ (\neg P(x) \vee \neg Q(x,y))$

c. $\forall y \ \exists x \ (\neg P(x) \vee \neg Q(x,y))$

d. $\forall x \ \exists y \ (\neg P(x) \vee \neg Q(x,y))$

6 Theorem: A group of 5 kids has a total of 12 chocolate bars. Then at least one of the kids has at least three chocolate bars.

A proof by contradiction of the theorem starts by assuming which fact?

a. All the kids have three or fewer chocolate bars.

*b. All the kids have less than three chocolate bars.

c. There is a kid with three or fewer chocolate bars.

d. There is a kid with r fewer than three chocolate bars.

7 Which statement is the contrapositive of: "If x = 4, then 3x = 12."

a. If $x = 4$, then $3x = 12$.

b. If $3x = 12$, then $x = 4$.

c. If $x \neq 4$, then $3x \neq 12$.

*d. $3x \neq 12$, then $x \neq 4$.

8 $A = \{x \in Z : x \text{ is even}\}$

$C = \{3, 5, 9, 12, 15, 16\}$

$D = \{5, 7, 8, 12, 13, 15\}$

Select the set corresponding to $C - (A \oplus D)$**.**

a. {3, 9, 16}

*b. {3, 9, 12}

c. {3, 5, 9, 15}

d. {3, 7, 8, 9, 13, 16}

9 What is the cardinality of the power set of {{∅}, 1, {2,3}}?

a. 6

b. 7

*c. 8

d. 9

10 Which of the following functions(Z->Z) is onto (surjective)?

a. f(x) = 4/x

b. f(x) = 2x

*c. f(x)=⌊(x+1)/2⌋

d. f(x) = $|x|^{-3}$

11 Determine which of the following f is a function from Z to R

a. $f(n) = \pm (2n + 3)$

b. $f(n) = \log_2(n)$

c. $f(n) = 1 / (n^2 - 1)$

*d. $f(n) = n / (n^2 + 1)$

12 Determine which of these functions is a bijection (one-to-one & onto) from R to R.

*a. $f(x) = 3x^3 + 4$

b. $f(x) = -3x^2 + 7$

c. $f(x) = x^2 + 1$

d. $f(x) = \lceil x/2 \rceil$

13 Select the expression that is equivalent to $\overline{x}$.

a. $(x + \overline{x})(y + x)$

b. $\overline{x}\, y + x$

c. $(\overline{x} + y)\overline{y}$

*d. $\overline{xy} + \overline{x}$

14 Select the description that characterizes the Boolean expression: $\overline{xyz}$

*a. Neither CNF nor DNF

b. CNF, but not DNF

c. DNF, but not CNF

d. CNF and DNF

15 Which of the following boolean equation is not satisfiable?

a. $(x + y)(x + z)(y + \overline{z})$

*b. $(\overline{x + y})(x + z)(y + \overline{z})$

c. $(\overline{x} + \overline{y})(x + z)(y + \overline{z})$

d. $(\overline{xy})(x + z)(y + \overline{z})$

16 Select the function that is $\Theta(n \log n)$.

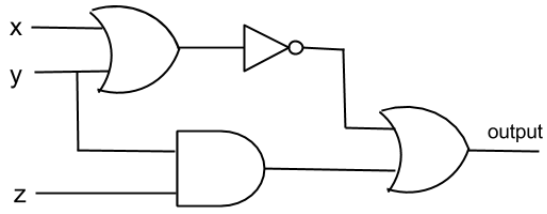
a. $5n + 17 \log n$

b. $6n \log n + n^{1.1} + 2$

*c. $23n \log(\log n) + 3n \log n$

d. $2^n \log n + n$

17 Select the Boolean expression that corresponds to the output of the Boolean circuit below:



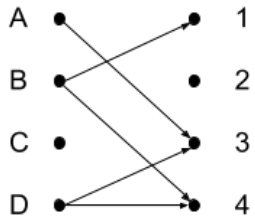
a. $\overline{x} + y + yz$

b. $x + y + \overline{yz}$

*c. $\overline{x + y} + yz$

d. $\overline{xy}(y + z)$

18 Select the set that corresponds to the relation given in the arrow diagram below:



*a. { (A, 3), (B, 1), (B, 4), (D, 3), (D, 4) }

b. { (A, 3), (B, 1), (B, 2), (D, 3), (D, 4) }

c. { (1, B), (3, A), (3, D), (4, B), (4, D) }

d. { (1, B), (2, B), (3, A), (3, D), (4, D) }

19 Recall that:

The relation R is ***reflexive*** if for every x ∈ A, xRx.

The relation R is ***symmetric*** if for every x,y ∈ A, xRy implies that yRx.

The domain of a relation R is the set of integers. xRy if $y = x^2$. Select the description that accurately describes relation R.

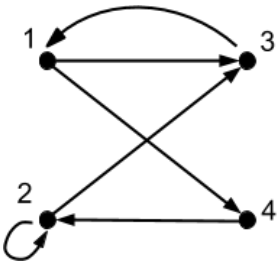
a. Reflexive

b. Anti-reflexive

c. Symmetric

*d. Anti-symmetric

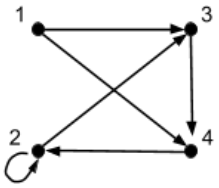
20 Graph G is defined by the arrow diagram below.



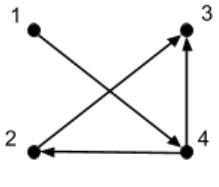
Select the pair of vertices such that there is no walk of length 4 in G from the first vertex to the second vertex.

a. 1, 3
*b. 1, 4
c. 2, 1
d. 4, 3

21 S and T are binary relations on the set {a, b, c, d} and are defined by the arrow diagrams below:



S



T

Select the pair that is not in S o T.
a. (1, 3)
b. (2, 3)
*c. (2, 4)
d. (4, 3)

22 Which relation on the set {1, 2, 3, 4} is a partial order?
a. { (1, 2), (2, 3), (1, 3), (4, 3) }
b. { (1, 2), (2, 3), (1, 3), (3, 4) }
c. { (1, 2), (2, 3), (1, 3), (3, 4), (1, 1), (2, 2), (3, 3), (4, 4) }
*d. { (1, 2), (2, 3), (1, 3), (4, 3), (1, 1), (2, 2), (3, 3), (4, 4) }

23 Which of the following is incorrect about strict orders?
a. A relationship xRy is called strict order if it's denoted by $x \prec y$
b. A relation R is a strict order if R is transitive and anti-reflexive
c. A strict order is basically a partial order without the self-loop
*d. Strict order doesn't needs to be anti-symmetric, while a partial order does

24 The domain of relation R is the set of all integers. xRy if $|x - y| \leq 1$. Which statement correctly characterizes the relation R?
a. R is an equivalence relation.
b. R is not an equivalence relation because R is not reflexive.
c. R is not an equivalence relation because R is not symmetric.
*d. R is not an equivalence relation because R is not transitive.

25 A = {a, b, c, d} X = {1, 2, 3, 4}.
Each choice defines a function whose domain is A and whose target is X. Select the function that has a well-defined inverse.
a. f = {(a, 3), (b, 4), (c, 3), (d, 4)}
b. f = {(a, 3), (b, 3), (c, 3), (d, 3)}
*c. f = {(a, 3), (b, 4), (c, 2), (d, 1)}

d. f = {(a, 3), (b, 4), (c, 2), (d, 4)}

26 Select the asymptotic worst-case time complexity of the following algorithm:

Input: A = [a_1, a_2,...,a_n], an array of ints.
n: the length of the sequence

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For i = 1 to n
  If( A[i] < x ) Return("True")
End-for
Return( "False" )
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a. $\Theta(1)$
*b. $\Theta(n)$
c. $\Theta(n^2)$
d. $\Theta(n^3)$

27 A = {1, 2, {3, 4}, {5, 6, 7}}, select the statement that is true.
a. $\{3\} \in A$
b. $\{3,4\} \subseteq A$
*c. $\{1,2\} \subseteq A$
d. $\{1,2\} \in A$

28 For the function n^k and c^n, what is the asymptotic relationship between these functions? Assume k >= 1, c > 1 and k, c are constants
*a. $n^k = O(c^n)$
b. $n^k = \Omega(c^n)$
c. $c^n = \Theta(n^k)$
d. $c^n = O(n^k)$

29 Select the set that is equivalent to $(B \cap C) \cup \varnothing$.
a. $\varnothing$
b. B
c. C
*d. $B \cap C$

30 The predicate T is defined as:
T(x, y, z): $(x+y)^2 = z$
Select the proposition that is true.
a. T(4, 1, 5)
*b. T(4, 1, 25)

c. T(1, 1, 1)
d. T(4, 0 2)